Radiation reaction in fusion plasmas

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The effects of a radiation reaction on thermal electrons in a magnetically confined plasma, with parameters typical of planned burning plasma experiments, are studied. A fully relativistic kinetic equation that includes the radiation reaction is derived. The associated rate of phase-space contraction is computed and the relative importance of the radiation reaction in phase space is estimated. A consideration of the moments of the radiation reaction force show that its effects are typically small in reactor-grade confined plasmas, but not necessarily insignificant.

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I. INTRODUCTION

A charged particle that is accelerated by an external electromagnetic field will radiate and, because of the radiation, be subject to an additional force called the radiation reaction force (RRF) [1]. Under ordinary circumstances the RRF is very small compared to the external force that induces it. Therefore it is rarely taken into account in studies of laboratory plasma. Recently, however, Andersson, Helander, and Eriksson [2] have shown that the radiation reaction can bear significantly on the behavior of runaway electrons in high-temperature tokamaks. Here we look at radiation reaction effects associated with thermal electrons in a reactor-sized tokamak such as ITER [3].

Our starting point differs from the previous analysis in using the relatively recent formulation of radiation reaction due to Rohrlich [4] (see also Ref. [5]). Beginning with the perturbative treatment of Landau and Lifshitz [6], Rohrlich derives an exact expression for the RRF that cures the conventional formulation of its major defects: (1) The Rohrlich version of the RRF vanishes in the absence of an external accelerating field. (2) The possibility of radiative runaway is avoided, and the time derivative of the acceleration does not appear. Thus the kinetic equation, whose derivation we outline in Sec. II, can be expressed in six-dimensional phase space.

A conclusion of Sec. II is that in the confined plasma case the RRF is most naturally compared, not to the macroscopic electromagnetic field, but rather to the dynamical friction due to Coulomb collisions. The point is that the RRF, like dynamical friction, does not conserve phase space. We find, not surprisingly, that under typical fusion conditions the dynamical friction exceeds the RRF by roughly three orders of magnitude. However, the RRF is distinctive in acting primarily on electrons. By depleting energy and momentum from the electron fluid exclusively, the RRF acts like a collisional process that fails to conserve plasma momentum and energy.

In Sec. III we explore this qualitative difference by examining the energy-momentum moment of the RRF and comparing it to the corresponding, well-known moments of the collision operator. Recalling that the outward particle flux due to collisions is automatically ambipolar in an axisymmetric confinement system, we compute the nonambipolar flux due to the RRF and discuss its significance. We also compare the electron energy loss due to the RRF to its collisional counterpart.

Our conclusions are summarized in Sec. IV.

II. KINETIC THEORY OF THE RADIATION REACTION

A. General form of the kinetic equation

For simplicity and generality we express the kinetic equation in Lorentz-covariant form. We use units in which the speed of light is unity, although factors of c are restored in certain key results for the sake of clarity. The Minkowski tensor is denoted by $\eta^{\mu\nu}$; Greek indices vary from 0 to 3, Roman indices from 1 to 3. We use the notation, for any four-vector a^μ,

$$a^\mu = (a^0, a^i) = (a^0, \boldsymbol{a}).$$

The momentum four-vector is

$$p^\mu = (p^0, m\gamma(v)\boldsymbol{v}),$$

where $\boldsymbol{v}$ is the ordinary three-vector velocity of a particle with mass m and $\gamma(v)=(1-v^2)^{-1/2}$. For a physical particle, the mass shell condition

$$p_\mu p^\mu = -m^2 \tag{1}$$

yields $\boldsymbol{p}^0 = E(\mathrm{p})$ on the mass shell, where

$$E(\boldsymbol{p}) \equiv \sqrt{\boldsymbol{p}^2 + m^2}.$$

Recall that the relativistic factor γ measures the particle energy

$$E = m\gamma. \tag{2}$$

We denote an arbitrary four-vector force by $F^\mu(x,p,t)$; it prescribes the change in momentum p^μ with respect to proper time τ according to Newton's law:

$$\frac{dp^\mu}{d\tau} = F^\mu. \tag{3}$$

Recall here that, if t is the time measured in the frame in which the particle has velocity $\boldsymbol{v}$, then

$$dt = \gamma d\tau. \tag{4}$$

Since the forces under consideration cannot affect the rest mass, Eq. (1) implies $p_\mu F^\mu = 0$ or

$$F^0 = \frac{\boldsymbol{p}\cdot\boldsymbol{F}}{E}. \tag{5}$$

The conventional (relativistic) distribution function $f(\boldsymbol{x},\boldsymbol{p},t)$ depends on only three momentum variables. However, it is sometimes convenient to consider an equivalent distribution

$$g(x,p) = \frac{\delta(p^0 - E)}{E} f(\boldsymbol{x},\boldsymbol{p},)t, \tag{6}$$

which depends on the four-momentum. It can be shown that both f and g are Lorentz scalars. Notice that

$$\int d^4p g(x,p) = \int \frac{d^3p}{E} f(\boldsymbol{x},\boldsymbol{p},t).$$

Using Eq. (5) one finds that

$$\frac{\partial}{\partial p^\mu}(F^\mu g) = \delta(p^0 - E)\frac{\partial}{\partial \boldsymbol{p}}\cdot\left(\frac{\boldsymbol{F}f}{E}\right). \tag{7}$$

We now turn to the kinetic equation. A manifestly covariant kinetic equation for the distribution g is

$$\frac{\partial}{\partial x^\mu}\left(\frac{p^\mu g}{m}\right) + \frac{\partial}{\partial p^\mu}(F^\mu g) = C,$$

where C represents the collision operator. The collision operator has the form

$$C = -\frac{\partial}{\partial p^\mu}\Gamma_C^\mu, \tag{8}$$

where Γ_C^μ is the collisional flow in velocity space. The kinetic equation for g is therefore

$$\frac{\partial}{\partial x^\mu}\left(\frac{p^\mu g}{m}\right) + \frac{\partial}{\partial p^\mu}(F^\mu g + \Gamma_C^\mu) = 0. \tag{9}$$

To obtain the kinetic equation for f, in six-dimensional phase space, we first restrict the collisional flow to the mass shell using

$$\Gamma_C^\mu = \frac{\delta(p^0 - E)}{E}\overline{\boldsymbol{\Gamma}}_C^\mu.$$

Since rest mass is conserved, Eq. (5) holds and we have

$$\frac{\partial}{\partial p^\mu}\Gamma_C^\mu = \delta(p^0 - E)\frac{\partial}{\partial \boldsymbol{p}}\cdot\left(\frac{\overline{\boldsymbol{\Gamma}}_C}{E}\right). \tag{10}$$

Now we combine Eqs. (6), (7), and (10) to find

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\frac{\boldsymbol{p}}{E}f\right) + \frac{\partial}{\partial \boldsymbol{p}}\cdot\left(\frac{m\boldsymbol{F}f + \overline{\boldsymbol{\Gamma}}_C}{E}\right) = 0. \tag{11}$$

The same result can be obtained starting from the microscopic (Klimontovich-Dupree) distribution function [7].

We next verify that our kinetic equation conserves particles. The four-vector particle flow is defined by

$$\Gamma^\mu \equiv \int d^4p p^\mu g = \int \frac{d^3p}{E} p^\mu f. \tag{12}$$

Its temporal component is the density, denoted by

$$\int d^3p f(\boldsymbol{x},\boldsymbol{p},t) = n(\boldsymbol{x},t). \tag{13}$$

The particle conservation law is obtained by integrating Eq. (11) over all three-momenta. We quickly find

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{\Gamma} = 0$$

or

$$\frac{\partial \Gamma^\mu}{\partial x^\mu} = 0. \tag{14}$$

The total electrodynamic force on a charged particle is

$$F^\mu = F_L^\mu + F_R^\mu. \tag{15}$$

Here the first term denotes the Lorentz force on a particle with charge e and mass m:

$$F_L^\mu = \frac{e}{m}F^{\mu\nu}p_\nu, \tag{16}$$

where $F^{\mu\nu}$ is the Faraday tensor. The second term F_R^μ is the RRF. Following Rohrlich [4] we express this force as

$$F_R^\mu = \frac{2}{3}\left(\frac{e}{m}\right)^3[p_\kappa p^\lambda \partial_\lambda F^{\mu\kappa} - eWe_\lambda^\kappa p^\lambda(\eta_\kappa^\mu + m^{-2}p_\kappa p^\mu)]. \tag{17}$$

Here $W = B^2 - E^2$ is the well-known Lorentz scalar and

$$e_\lambda^\kappa = -\frac{F^{\kappa\nu}F_{\nu\lambda}}{W} \tag{18}$$

is an operator introduced previously [8]. The key property of e_λ^κ applies when the parallel electric field is relatively small,

$$\boldsymbol{E}\cdot\boldsymbol{B} \ll W,$$

as is the case in tokamak experiments. Then one finds [8] that e_λ^κ acts as a projection operator for the two directions perpendicular to $\boldsymbol{B}$:

$$e_\lambda^\kappa A^\lambda \approx \boldsymbol{A}_\perp$$

for an arbitrary four-vector A^μ.

It can be verified that the RRF, like the Lorentz four-force, satisfies Eq. (5). The details are omitted.

B. Collision term

The collisional flow $\overline{\boldsymbol{\Gamma}}_C$ results from Coulomb collisions. In a hot, weakly coupled plasma it has Fokker-Planck form

$$\overline{\boldsymbol{\Gamma}}_C = -\boldsymbol{D}\cdot\frac{\partial f}{\partial \boldsymbol{p}} + \boldsymbol{F}_C f, \tag{19}$$

where $\boldsymbol{D}$ is the diffusion tensor and $\boldsymbol{R}$ is the dynamical friction. After substituting this form into Eq. (11) we have

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{E} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\partial}{\partial \boldsymbol{p}} \cdot \left[\frac{m}{E} f(\boldsymbol{F}_L + \boldsymbol{F}_R + \boldsymbol{F}_C) \right] = \frac{\partial}{\partial \boldsymbol{p}} \cdot \boldsymbol{D} \cdot \frac{\partial f}{\partial \boldsymbol{p}}. \tag{20}$$

The radiation reaction also acts like a friction force, so our comparisons, below, will involve $\boldsymbol{F}_R$ and $\boldsymbol{F}_C$. Here we recall the form of $\boldsymbol{F}_C$ in the case of Maxwellian, or nearly Maxwellian, electrons:

$$\boldsymbol{F}_C = \frac{2\pi e^4 n \log \Lambda}{m_e} \frac{\boldsymbol{v}}{v^3} \left[\operatorname{erf}(x) - x \frac{d\operatorname{erf}(x)}{dx} \right], \tag{21}$$

where $x \equiv v/v_{te}$, with $v_{te} = \sqrt{2T_e/m_e}$ the electron thermal speed. The quantity $\log \Lambda$ is the Coulomb logarithm.

C. Phase-space contraction

The quantity $\boldsymbol{F}_L$, constituting the three spatial components of a Lorentz four-vector, differs from the ordinary Lorentz force by a factor of $\gamma = E/m$. It satisfies

$$\frac{\partial}{\partial \boldsymbol{p}} \cdot \left(\frac{\boldsymbol{F}_L}{E} \right) = 0. \tag{22}$$

Since the invariant phase-space volume element is $d^3x d^3p/E$, Eq. (22) expresses the conservation of phase space under the Lorentz force and allows the Lorentz force in Eq. (11) to be extracted from the momentum-divergence operator. Indeed the kinetic equation is conventionally presented with the macroscopic force outside the momentum derivative, thereby distinguishing this force from collisional effects. The latter do not conserve phase space in so far as they remove energy and momentum from a given plasma species.

Here we point out that the total electrodynamic force does not conserve phase space. The radiation reaction causes particles to lose energy and momentum, even in the absence of collisions. We next compute the resulting contraction of phase space.

First note that

$$\frac{\partial}{\partial p^\mu} (p_\kappa p^\lambda) = p_\kappa \eta^\lambda_\mu + p^\lambda \eta_{\kappa\mu}. \tag{23}$$

Therefore, if we introduce for convenience Z^μ such that

$$F_L^\mu = -\frac{2}{3} \left(\frac{e}{m} \right)^3 Z^\mu,$$

then

$$\begin{aligned} \frac{\partial Z^\mu}{\partial p^\mu} &= p_\kappa \partial_\mu F^{\mu\kappa} - eW e^\kappa_\mu (\eta^\mu_\kappa + m^{-2} p_\kappa p^\mu) - 5m^{-2} eW e^\kappa_\lambda p^\lambda p_\kappa \\ &= p_\kappa \partial_\mu F^{\mu\kappa} - 2eW - 6m^{-2} eW e^\kappa_\lambda p^\lambda p_\kappa. \end{aligned} \tag{24}$$

Maxwell's field equations convert the first term of Eq. (24) to

$$p_\kappa \partial_\mu F^{\mu\kappa} = -p_\kappa J^\kappa,$$

where J^κ is the four-vector current density. According to Eq. (18) the last term is

$$\begin{aligned} -6m^{-2} eW e^\kappa_\lambda p^\lambda p_\kappa &= 6m^{-2} F^{\kappa\sigma} p_\kappa F_{\sigma\lambda} p^\lambda \\ &= -6m^{-2} F^{\sigma\kappa} p_\kappa F_{\sigma\lambda} p^\lambda \\ &= -6m^{-2} (m/e)^2 F_L^\sigma F_{L\sigma}, \end{aligned}$$

which is (proportional to) the square of the Lorentz force. We conclude that

$$\frac{\partial F_R^\mu}{\partial p^\mu} = -\frac{2}{3} \left(\frac{e}{m} \right)^3 (p_\kappa J^\kappa + 2eW + 6e^{-2} F_L^\sigma F_{L\sigma}). \tag{25}$$

As a damping mechanism, the RRF cannot be expected to conserve phase space; the result, Eq. (25), is unsurprising. However, it confirms that the proper reference in estimating the size and importance of the RRF is not the enormously larger Lorentz force, but dynamical friction.

D. Relative size

We compare typical magnitudes of the RRF and dynamical friction in a parameter regime characterizing a large tokamak such as ITER. In such a device the electron thermal speed is roughly a tenth of the light speed, so we are interested in the nonrelativistic limit of the RRF. It is easily seen from Eq. (17) that

$$\boldsymbol{F}_R = -\frac{2}{3} \frac{e^4}{m_e^2} B^2 \boldsymbol{v}_\perp + O(v^2) \tag{26}$$

is the dominant contribution when $B \gg E$ and $v \ll 1$. (Here and below e represents the magnitude of the electronic charge.) For the dynamical friction we use Eq. (21) to estimate

$$\boldsymbol{F}_C \approx \frac{4\pi e^4 n \log \Lambda}{m_e v_{te}^3} \boldsymbol{v}_\perp \tag{27}$$

for thermal electrons. When computing the ratio $|F_R/F_C|$ we return to conventional units, with the light speed $c = 3 \times 10^{10}$ cm/sec. One finds

$$\frac{F_R}{F_C} \sim \frac{m_i}{m_e \log \Lambda} \left(\frac{v_A}{c} \right)^2 \left(\frac{v_{te}}{c} \right)^3, \tag{28}$$

where $v_A = B/\sqrt{4\pi m_i n}$ is the Alfvén speed. In a reactor-regime tokamak, each velocity ratio is of order 10^{-1} and $\log \Lambda \sim 20$, whence

$$\frac{F_R}{F_C} \sim 10^{-3}. \tag{29}$$

Recall that the lowest-order distribution function in a magnetically confined plasma is independent of gyrophase and therefore unaffected by the Lorentz magnetic force [9]. In fact it is determined by a balance between collisional diffusion and dynamical friction. The radiation reaction combines with dynamical friction and therefore modifies the lowest-order distribution; Eq. (29) shows that for thermal particles the modification is minor.

The macroscopic properties of the confined plasma depend upon moments of the kinetic equation. We next consider how these properties are affected by the radiation reaction.

III. MOMENTS OF THE RADIATION REACTION

A. Energy-momentum evolution

After multiplying the kinetic equation (11) by p^μ and integrating over momentum [using the mass-shell Jacobian as in Eq. (12)], we obtain a familiar energy-momentum evolution law. Specializing to the case of electrons, but suppressing e subscripts, we have

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} + eF^{\mu\nu}\Gamma_\nu = \mathcal{C}^\mu - \mathcal{R}^\mu, \tag{30}$$

where $T^{\mu\nu}$ is the electron energy-momentum tensor, $\mathcal{C}^\mu$ is the four-momentum moment of the collision operator,

$$\mathcal{C}^\mu = \int d^4 p p^\mu C, \tag{31}$$

and $\mathcal{R}^\mu$ is the corresponding moment of the RRF:

$$\mathcal{R}^\mu = \int d^4 p p^\mu \frac{\partial F_R^\nu g}{\partial p^\nu}. \tag{32}$$

The three spatial components of Eq. (30) describe the acceleration of the electron fluid; the temporal component describes the electron energy change.

We can use Eq. (8) to write the collisional moment as

$$\mathcal{C}^\mu = \int d^4 p \Gamma_C^\mu = \int \frac{d^3 p}{E}\overline{\Gamma}_C^\mu. \tag{33}$$

Similarly,

$$\mathcal{R}^\mu = -\int d^4 p F_R^\mu g = -\int \frac{d^3 p}{E} F_R^\mu f. \tag{34}$$

The moments of the collision operator are well known [10] and considered presently. A complete expression for the RRF moments may be found in the recent literature [11]; here we are interested only in the nonrelativistic limit, where Eq. (26) quickly yields

$$\mathcal{R}^k \approx \frac{2}{3}\frac{e^4}{m^2} W e^{k\nu}\Gamma_\nu.$$

Notice that the dominant temporal component, to be considered presently, comes from a different term in F_R. Since W is well approximated by B^2 in a magnetic confinement device, we have

$$\boldsymbol{\mathcal{R}} = \frac{2}{3}\frac{e^4}{m^2}B^2\boldsymbol{\Gamma}_\perp. \tag{35}$$

B. Comparison to collisional friction

The spatial components of the collisional moment give the collisional friction force, measured by Ohmic current [10]. In view of Eq. (35), the perpendicular component is the appropriate standard of reference:

$$\boldsymbol{\mathcal{C}} \sim \frac{en\boldsymbol{J}_\perp}{\sigma_\perp},$$

where $\boldsymbol{J}_\perp$ is the perpendicular plasma current density and

$$\sigma_\perp \sim e^2 n\tau_{ei}/m_e$$

is the perpendicular conductivity. Here τ_{ei} is the electron-ion collision time. Since lowest-order fluid equilibrium provides the diamagnetic current $\boldsymbol{J} \sim \boldsymbol{\nabla} p/B$, with $p = n(T_e + T_i)$ the plasma pressure, we can estimate

$$\boldsymbol{\mathcal{C}} \sim \frac{m_e \boldsymbol{\nabla} p}{eB\tau_{ei}}. \tag{36}$$

Turning to the corresponding RRF of Eq. (35), we note that $\boldsymbol{\Gamma}_\perp$ has a corresponding diamagnetic term $\boldsymbol{\Gamma}_\perp \sim \boldsymbol{\nabla}(nT_e)/eB$. Hence the ratio $\mathcal{R}_\perp/\mathcal{C}_\perp$ is easily found to be

$$\frac{\mathcal{R}_\perp}{\mathcal{C}_\perp} \sim \xi,$$

where

$$\xi \equiv \Omega_e^2 \tau_{ei}(r_0/c) \tag{37}$$

is the basic parameter measuring the significance of the RRF on transport phenomena. Here we have restored factors of c, as in Eq. (28) and assumed $T_i \sim T_e$; $\Omega_e = eB/m_e c$ is the electron gyrofrequency and $r_0 = e^2/m_e c^2$ is the classical electron radius.

Note that

$$r_0/c \approx 10^{-23}\ \text{sec}.$$

The presence of this very small factor explains why the RRF is ordinarily ignored in confinement physics. In the present case, it is almost defeated by the gyrofrequency factors, since

$$\Omega_e \sim 10^{12}\ \text{sec}^{-1}$$

in devices like ITER. However, the collision time is roughly $\tau_{ei} \sim 3 \times 10^{-4}$, so

$$\xi \sim 3 \times 10^{-3} \tag{38}$$

remains small. We next show that even this small a force can play a significant role in confinement.

C. Nonambipolar electron flux

Lowest-order equilibrium of the electron fluid in a confined plasma depends upon a balance among the electron pressure gradient, the fluid Lorentz force, and collisional friction. Here we add the RRF to this balance and obtain the equilibrium equation

$$\boldsymbol{\nabla} p_e + en(\boldsymbol{E} + \boldsymbol{V}_e \times \boldsymbol{B}) = \boldsymbol{\mathcal{C}}_e - \boldsymbol{\mathcal{R}}.$$

Of course the last term on the right-hand side is very small. We first neglect this term and then solve for the perpendicular velocity to obtain a familiar result

$$\boldsymbol{V}_{e\perp} \approx \boldsymbol{V}_E + \boldsymbol{V}_{eP} + \boldsymbol{V}_{eC}, \tag{39}$$

where $\boldsymbol{V}_E$ is the $E \times B$ drift, $\boldsymbol{V}_{eP} = \boldsymbol{b} \times \boldsymbol{\nabla} p_e/(en)$ is the diamagnetic drift, and $\boldsymbol{V}_{eC} = \boldsymbol{b} \times \boldsymbol{\mathcal{C}}_e/(en)$ yields classical collisional transport. Here $\boldsymbol{b} = \boldsymbol{B}/B$ is a unit vector in the direction of the confining field.

Let us denote the small gyroradius parameter by

$$\delta \equiv (\text{gyroradius})/(\text{gradient scale length}).$$

Then one finds that $\boldsymbol{V}_E$ and $\boldsymbol{V}_{eP}$ are measured by δv_{te} while $\boldsymbol{V}_{eC} \sim \delta^2$. Indeed a straightforward calculation of the friction force provides [9]

$$\boldsymbol{V}_{eC} = -(m_e \Omega_e^2 n \tau_{ei})^{-1} \boldsymbol{\nabla} p. \tag{40}$$

Because collisions conserve momentum,

$$\boldsymbol{\mathcal{C}}_e = -\boldsymbol{\mathcal{C}}_i,$$

classical transport is intrinsically ambipolar:

$$\boldsymbol{V}_{iC} = \boldsymbol{V}_{eC}.$$

In order to compute an ambipolar potential, one must proceed to third order in δ_i, where nonambipolar ion flows appear. Requiring ambipolarity at third order indeed yields an equation for the potential (see, for example, [12]). Note, however, that

$$\delta_i \sim 10^{-3} \tag{41}$$

in a large tokamak. At this level a number of small effects can enter.

The RRF is an example of such an effect. By adding to the friction force it contributes an additional electron flux

$$\boldsymbol{V}_R = \boldsymbol{b} \times \boldsymbol{\mathcal{R}}/(en),$$

which is strictly "unipolar," since there is no measurable RRF effect on ions. We calculate this flow iteratively from Eq. (35), using the lowest-order velocity $\boldsymbol{V}_{e\perp}^{(0)} \equiv \boldsymbol{V}_E + \boldsymbol{V}_{eP}$:

$$\boldsymbol{V}_R = -\frac{2}{3}\Omega_e r_0 \boldsymbol{b} \times \boldsymbol{V}_{e\perp}^{(0)}. \tag{42}$$

As expected, the new flow is small compared to classical transport:

$$\frac{V_R}{V_C} \sim \xi, \tag{43}$$

as in Eq. (37). However, Eqs. (38) and (41) show that the flow due RRF is not necessarily small compared to the ion flow that is used to compute the ambipolar potential.

As a concrete example, we consider a fusion experimental device on the scale of the planned international experiment, ITER. Thus we assume a magnetic field $B = 5.3$ T and an electron-ion collision time $\tau_{ei} = 9.5 \times 10^3$ sec to find

$$\xi = 9.2 \times 10^{-4}.$$

In the same device one finds that the parameter δ_i has the nominal value 1.2×10^{-3}. Hence the radiation reaction force contributes to nonambipolar diffusion at a level roughly 70% the size of conventional contributions.

D. Energy loss

Electrons in a tokamak lose energy through several channels, including collisional exchange with ions, line radiation, and bremstrahlung. Here we compare the energy loss from radiation reaction to the collision exchange rate. The latter is given by a familiar formula [10]

$$Q_{ei} = 3\frac{m_e}{m_i}\frac{n}{\tau_{ei}}(T_e - T_i). \tag{44}$$

The energy loss due to the RRF, $\mathcal{R}^0$, comes primarily from the last term in Eq. (17). The term that dominates $\boldsymbol{\mathcal{R}}$ contributes negligibly here because the projector e_λ^0 is very small in fusion experiments. Hence, using Eq. (34) and recalling that the electrons are Maxwellian in lowest order, we have

$$\mathcal{R}^0 = -\frac{2}{3}\frac{e^4}{m_e^3}B^2 \int \frac{d^3p}{E} f_M v_\perp^2, \tag{45}$$

where f_M is the electron Maxwellian normalized as in Eq. (13). The integral is elementary and we find

$$\mathcal{R}^0 = -\frac{4}{3}\frac{e^4}{m_e^3}B^2 p_e. \tag{46}$$

From Eqs. (44) and (46) we compute

$$\left|\frac{\mathcal{R}^0}{Q_{ei}}\right| \sim \frac{m_i}{m_e}\xi\frac{T_e}{T_e - T_i}. \tag{47}$$

Here ξ appears multiplied by the large mass ratio, yielding a quantity that is no longer small: radiation reaction energy loss competes, in the environment of a large, hot, magnetically confined plasma, with electron-ion energy exchange. Equally significant is the fact that even after the electrons and ion have equilibrated, an electron energy loss rate comparable or even larger than the equilibration rate is sustained by RRF. This must be taken into account for a proper energy inventory.

To consider again a specific example, we note that ITER temperatures are expected to satisfy

$$\frac{T_e}{T_e - T_i} = 0.09,$$

whence

$$\left|\frac{\mathcal{R}^0}{Q_{ei}}\right| \sim \frac{m_i}{m_e}\xi\frac{T_e}{T_e - T_i} = 18.5.$$

Thus radiation reaction losses will be nearly 20 times those due to collisional energy exchange in ITER.

IV. SUMMARY

Because of the radiation reaction force, classical electrodynamics does not conserve phase space. The damping experienced by a radiating electron in a hot plasma is similar to the dynamical friction from collisions with ions, with two key differences: it is smaller than the collisional process by a factor of order 10^{-3} (in fusion experimental regimes), and it fails to conserve plasma momentum. Because the moments of the energy and momentum losses are properly compared to collisional effects that are themselves quite small, the radiation reaction can affect fusion plasma confinement in measurable ways.

The most important effects of the radiation reaction may pertain to suprathermal electrons, such as runaway electrons

[2] or electrons on the tail of a Maxwellian distribution. Thus, while our analysis focuses on thermal electrons, we develop a fully relativistic kinetic description in Sec. II that could be applied more generally. The kinetic equation (20) makes the radiative contraction of phase space explicit, as in Eq. (25). It also shows the parallel between the radiation reaction and dynamical friction, leading to an estimate, Eq. (28), of the relative importance of radiation reaction effects in the kinetic theory of a fusion-regime plasma.

A similar comparison between the momentum loss due to radiation and the momentum exchanged by electron-ion collisions is shown in Sec. III to be characterized by the parameter

$$\xi = \Omega_e^2 \tau_{ei}(r_0/c),$$

which is of order 10^{-3} in a device like ITER. Effects this small are rarely of interest to magnetic confinement physics. However, we note that the ratio between nonambipolar ion radial transport (third order in the gyroradius) and conventional, ambipolar transport is comparable to ξ. (The occurrence of 10^{-3} in three contexts—kinetic effects of the radiation reaction, its transport effects, and the ion gyroradius parameter—is coincidental.) Therefore the radiation reaction should be included in classical calculations of the ambipolar potential. Finally we show that the electron energy loss due to the radiation reaction is comparable, under fusion-regime parameters, to the collisional energy exchange with ions.

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